Received: *February 19, 2001*, Accepted: *March 26, 2001*

Living inside the horizon of the D3-brane

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Abstract: We consider a brane world residing in the interior region inside the horizon of the D3-brane. The horizon size can be interpreted as the compactification size. The macroscopically large size of extra dimensions then can be derived from the underlying string theory that has only one physical scale, i.e. the string scale. Then, the hierarchy between the string scale and the Planck scale is provided by Ramon-Ramon charge of the D3-brane. This picture also offers a new perspective on various issues associated with the brane world scenarios including the cosmological constant.

Keywords: Extra Large Dimensions, D-branes, Classical Theories of Gravity.

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Contents

1. Introduction

String theory spacetimes with conserved quantum numbers are commonly black p-branes [1]. In particular, the extremal p-brane with R-R charge can be described as a Dp-brane [2]. The discovery of D-brane drastically changed the conventional viewpoint that the standard model fields and the graviton should propagate in the same spacetime. The standard model fields can be naturally confined on D-brane, while the graviton can freely propagate through the full spacetime. Recently, this concept has been extensively introduced in approaches to resolving the hierarchy problem and getting a small cosmological constant without relying on supersymmetry in the framework of brane world scenarios.

In refs. [3]–[6], it was shown that the string scale l_s^{-1} could be reduced from the four-dimensional Planck scale, $M_{pl} \sim 10^{19}\,\mathrm{GeV}$, to the electroweak scale, $m_{EW} \sim 10^3\,\mathrm{GeV}$, by introducing large extra dimensions. However, this scenario has the problem of stabilization of the large extra dimensions or, equivalently, dynamical determination of the large size of extra dimensions in a theory whose fundamental length is the inverse weak scale.

Subsequently, Randall and Sundrum (RS) proposed a scheme in which a five-dimensional spacetime contains strongly gravitating 3-branes which produce a warped or nonfactorizable geometry, e.g. a slice of AdS_5. In their first model (RSI) with two 3-branes [7], solving the hierarchy problem does not require large extra dimensions. In the second model (RSII) [8] which has only one 3-brane of positive tension, RS also showed that, even for an infinite extra dimension, the effective gravity theory on the brane can be still four-dimensional and not five-dimensional up to leading behavior, provided that the embedding spacetime has a very particular curvature.

Many attempts to resolve the cosmological constant problem have been made within the brane world scenarios [9]–[16]: If our four-dimensional world is embedded in a higher-dimensional spacetime, the changes of the vacuum energy of the brane, the brane tension, may affect only the curvature in the extra dimensions, keeping the Poincaré invariance of the four-dimensional worldvolume.

Various attempts have been made to realize the Randall-Sundrum scenario within 5-dimensional supergravity theories [17]–[21] and a compactified string or M-theory [22]–[24]. In particular, the authors of ref. [17] investigated singular, supersymmetric domain-wall solutions supported by the massive breathing mode scalars of reductions in M-theory or string theory. The spacetime on one side of such a wall asymptotes to the Cauchy horizon of the anti-de Sitter (AdS) spacetime, while there is a naked singularity on the other side. The higher dimensional interpretation of these domain wall solutions is given as the interior region between the singularity and the horizon of the higher-dimensional non-dilatonic p-branes [17, 19]. This implies that the interior region could provide a setup for brane world scenarios, even though it is normally excluded from the maximal analytic extension.

On the other hand, the authors of ref. [25] considered an extremal black hole-like global defect which was called as 'black global p-brane', and showed that the interior region inside the horizon of the black brane possesses all of features needed for a realistic brane world scenario. In the picture of ref. [25], the size of the horizon can be interpreted as the compactification size, even though the interior region inside the horizon infinitely extends. And the large mass hierarchy is generated from topological charge of the brane, from which the horizon size is determined. In this picture, the change of the brane tension can forcefully be diluted via the Hawking radiation, because the change of the brane tension convert the bulk geometry of the extremal black brane into that of a non-extremal one, which evolves back to the extremal one through the Hawking radiation process.

In this paper, we consider the D3-brane living in the ten-dimensional spacetime and focus on the interior region inside the horizon of the D3-brane. The interior region has interpolation between the naked singularity and the near horizon region which is the infinitely long AdS throat ($AdS_5 \times S^5$). If the singularity is smoothed out by stringy effects, the physics will be close to that of the global black brane of ref. [25]. The interior region of D3-brane then can be interpreted as four-dimensional spacetime producted by six extra dimensions compactified to the size of the horizon, in that the four-dimensional Planck scale M_{pl} is determined by the ten-dimensional Planck scale $M_* \sim g_s^{-1/4} l_s^{-1}$ and the horizon size r_H via the relation $M_{pl}^2 \sim M_*^8 r_H^6$, and the gravity on a 3-brane residing in the interior region behaves as expected in a world with six extra dimensions compactified to size r_H. As the result of such interpretation, the compactification scale can be dynamically determined from the underlying string theory which has only one physical scale, i.e. string scale l_s^{-1}. A large mass hierarchy is then simply translated into large horizon size. Moreover, since the size of the horizon is determined by the Ramon-Ramon (R-R) charge of D3-brane, the large mass hierarchy then follows by taking large R-R charge and its stabilization is guaranteed by the conserved nature of the R-R charge. The observed extremely small cosmological constant could be explained provided that our world brane is embedded in the interior region of a very near-extremal D3-brane.

2. The spacetime of D3-brane

In string theory, the Dp-brane is a $(p+1)$-dimensional hyperplane in spacetime where open strings can end [2]. By the worldsheet duality, this means that the D-brane is also a source of closed strings. If there are N overlapping branes, open strings with endpoints on different branes correspond to $\mathrm{U}(N)$ gauge fields localized in the world-volume of Dp-brane. On the other hand, gravity corresponds to closed strings and these can move on the whole 10-dimensional spacetime.

On the other hand, it is believed that the Dp-brane and the extremal black p-brane in supergravity are two different descriptions of the same object. For Dp-branes with $p \neq 3$, the horizon and the singularity coincide, so there is null singularity. Moreover, the dilaton either diverges or vanishes at the horizon. While, for the D3-brane, the dilaton is constant and the horizon is regular. Together with these properties, since the D3-brane has a (3+1)-dimensional world volume, it may provide an interesting and concrete model for attempts to describe our universe as a 3-brane embedded in a higher-dimensional spacetime.

The supergravity solution of D3-brane with N units of R-R charge is given by

$$ds^2 = \sqrt{\sigma\Delta(r)}\ \eta_{\mu\nu}dx^\mu dx^\nu + \Delta(r)^{-2}\ dr^2 + r^2 d\Omega_5^2\,, \tag{2.1}$$

in terms of an appropriately defined Schwarzschild-type coordinate r, where

$$\Delta(r) \equiv 1 - \left(\frac{r_H}{r}\right)^4 \qquad \text{with} \qquad r_H^4 \equiv 4\pi l_s^4 g_s N\,, \tag{2.2}$$

where l_s is the string length scale and g_s is the string coupling. Here $\sigma = -1$ for the interior solution ($r < r_H$) and $\sigma = 1$ for the exterior solution ($r > r_H$). The spacetime has a horizon at $r = r_H$ and this horizon is regular. And the singularity at $r = 0$ is covered with the horizon. As discussed in [26], it is known that the metric (2.1) can be analytically extended through the Cauchy horizon at $r = r_H$ and out into another region. This can be seen by introducing a new radial coordinate ω by the relation

$$\omega = [\sigma\Delta(r)]^{1/4} \quad \Leftrightarrow \quad r = r_H\left(1 - \sigma\omega^4\right)^{-1/4}\,. \tag{2.3}$$

Since r is an analytic function of ω on the horizon at $\omega = 0$, the metric can be extended to negative values of ω. The extension is isometric to the original regions because the metric is invariant under $\omega \to -\omega$. For the exterior solution, repeating the continuation, one obtains a completely non-singular maximal analytic extension of the metric (2.1). The interior region is excluded from this extension. This is the case normally treated in the literatures. However, we have no reason to exclude the interior region from discussions given our ignorance concerning the singularity. In fact, there must be a R-R charged δ-function source at $r = 0$ because the D3-brane is coupled to the self-dual 5-form field strength and carry both "electric" and "magnetic" charge. The interior region then can be also analytically extended through the horizon to another region isometric to the original interior region excluding the exterior region.

As well known, the near horizon geometry is the infinitely long AdS throat. To see this, it will be useful to introduce a new radial coordinate ρ by

$$\rho = \int^r [\sigma\Delta(r')]^{-5/4} \; dr' \,, \tag{2.4}$$

where ρ runs from 0 to ∞ for the interior solution and from $-\infty$ to ∞ for the exterior solution. The horizon is now located at $\rho = \infty$ for the interior solution and at $\rho = -\infty$ for the exterior solution. With this new radial coordinate, the metric is then written as

$$ds^2 = \sqrt{\sigma\Delta(r(\rho))} \; \left(\eta_{\mu\nu}dx^\mu dx^\nu + d\rho^2\right) + r(\rho)^2 d\Omega_5^2 \,, \tag{2.5}$$

where the original radial coordinate r is given by a function of ρ and its asymptotic behaviors can easily be found. In the near horizon region, we can approximate the metric as

$$ds^2 \approx \frac{r_H^2}{\rho^2} \left(\eta_{\mu\nu}dx^\mu dx^\nu + d\rho^2\right) + r_H^2 d\Omega_5^2 \,, \tag{2.6}$$

which is the geometry of $AdS_5 \times S^5$ with curvature scales r_H^{-1}. This solution is an example of warped metric due to the backreaction of the brane.

So far the black 3-brane have been treated by using the classical supergravity. This description is appropriate when the curvature of the 3-brane geometry is small compared to the string scale, so that the stringy corrections are negligible. For the exterior region, since the curvature is bounded by r_H^{-1}, this requires $r_H \gg l_s$. To suppress string loop corrections, the effective string coupling g_s also needs to be kept small. Since the dilaton is constant, we can make it small everywhere in the D3-brane geometry by setting $g_s < 1$, namely $l_P < l_s$. That is, the supergravity approximation is valid everywhere when $l_p < l_s \ll r_H$ for the exterior solution. Since r_H is related to the R-R charge N as $r_H^4 = 4\pi l_s^4 g_s N$, this can also be expressed as $1 \ll g_s N < N$. On the other hand, since in the interior region the curvature is not bounded at the origin, the supergravity description will be valid only in a limited region. Because the curvature diverges as $\sim r_H^4/r^5$ near the singularity, the interior solution is valid within the region satisfying the condition $r \gg (4\pi g_s N)^{1/5} l_s$ and the stringy description might be needed in a region where $r \lesssim (4\pi g_s N)^{1/5} l_s$. In terms of the radial coordinate ρ, this corresponds to the region where $\rho \lesssim (4\pi g_s N)^{-1/20}\, l_s \lesssim l_s$. This implies that the singularity could be an artifact of the long-range supergravity approximation. The singularity is expected to be cured by stringy effects. Essentially the source for the R-R field strength is sitting at the singularity, i.e. the field energy density diverges as $\sim 1/r^{10}$. Since only the known source for the R-R 5-form field strength is D3-brane, we are naturally expected to see a stack of D3-branes when we probe the singularity with energy over the string scale.

3. Metric fluctuations and the singularity

Either the interior or the exterior region by itself does not seem to provide a suitable framework for a Randall-Sundrum configuration, because in one direction one may reach an AdS horizon, in the other direction one will either run into a singularity or on out into an unbounded flat space. However, as discussed in [17] in the context of the five-dimensional domain wall solution to the dimensionally reduced type IIB theory on S^5, such singular region by itself can provide a suitable framework for a Randall-Sundrum configuration, provided that the singularity is regularized.

The existence of the massless graviton state is evident because the theory always allows solutions with a general Ricci-flat metric $\bar{g}_{\mu\nu}(x)$, which satisfies the four-dimensional vacuum Einstein equation $\bar{R}_{\mu\nu}(\bar{g}) = 0$, instead of $\eta_{\mu\nu}$ in the metric (2.1), and the massless four-dimensional graviton is simply the usual gravitational wave solution of linearized four-dimensional vacuum Einstein equation. The Planck scale is finite in spite of the existence of the singularity in the interior region. One can easily see this examining the four-dimensional effective action. The four-dimensional Planck scale is given in terms of the 10-dimensional Planck scale by:

$$M_{pl}^2 = M_*^8 \int d\Omega_5 \int_0^{r_H} dr\; r^5 [-\Delta(r)]^{-1/2} = M_*^8 \cdot \frac{\pi^3}{3} r_H^6 \,, \tag{3.1}$$

where M_* ($\equiv (8\pi^6 g_s^2 l_s^8)^{-1/8}$) is the 10-dimensional Planck scale. $(\pi^3/3) r_H^6$ is just 5/8 times the volume of 6-dimensional sphere, so the radial direction has the effective size of $(5/8) r_H$.

Eq. (3.1) says that the four-dimensional Planck scale is determined by the ten-dimensional Planck scale and the horizon size via the familiar relation $M_{pl}^2 \sim M_*^8 r_H^6$ as in the usual Kaluza-Klein theories. This implies that the horizon size r_H could be interpreted as the effective size of 6 compact extra dimensions, even though the interior region infinitely extends.[1] This interpretation could be cleared up through a complete analysis of the effective 4D gravity.

[1] The six extra dimensions are a direct product of five-dimensional sphere S^5 with radius r_H and an infinitely extended radial direction. Despite being noncompact, due to the warped spacetime geometry, the radial direction yields a finite effective size $\sim r_H$.

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However, we should be careful in discussion of the graviton state because we do not have a complete understanding of the singularity. For a metric of the form of eq. (2.1), the gravitational fluctuations are given by replacing $\eta_{\mu\nu}$ with $\eta_{\mu\nu}+h_{\mu\nu}(x,y)$ where $h_{\mu\nu}$ is small compared with $\eta_{\mu\nu}$. With the gauge condition $\partial^\mu h_{\mu\nu}=0$, the linear fluctuations satisfy the covariant wave equation [30]:

$$\frac{1}{\sqrt{g}}\partial_M\left(\sqrt{g}\, g^{MN}\partial_N h_{\mu\nu}\right)=0\,, \tag{3.2}$$

where g_{MN} is the ten-dimensional background metric. Using the coordinates of the metric (2.1) and expanding $h_{\mu\nu}(x,r,\Omega)=\epsilon_{\mu\nu}e^{ip\cdot x}R_\ell(r)Y_\ell(\Omega)$, where Ω denotes angular variables, we have an equation for the radial wavefunction $R_{m\ell}(r)$:

$$-\frac{1}{r^5}\frac{d}{dr}\left(\Delta(r)^2\, r^5\frac{d}{dr}\right)R_{m\ell}(r)+\frac{\ell(\ell+4)}{r^2}R_{m\ell}(r)=m^2R_{m\ell}(r)\,, \tag{3.3}$$

where $\epsilon_{\mu\nu}$ is a constant polarization tensor and m $(=\sqrt{-p\cdot p})$ is the mass of the KK modes. The second term is the repulsive centrifugal potential which comes from the contribution of angular momentum. As easily expected, this equation is singular at $r=0$. Since this equation has the asymptotic form near the singularity:

$$-\frac{r_H^8}{r^8}\frac{d^2R_{m\ell}}{dr^2}+3\frac{r_H^8}{r^9}\frac{dR_{m\ell}}{dr}+\ell(\ell+4)\frac{R_{m\ell}}{r^2}=m^2R_{m\ell}, \tag{3.4}$$

the equation (3.3) requires regular solutions to behave near the singularity as follows

$$R_{m0}(r)\to a_0\left(1-\frac{m^2}{60r_H^8}r^{10}\right)+\cdots\,,\qquad \text{for } \ell=0\,, \tag{3.5}$$

$$R_{m\ell}(r)\to r^n \quad \text{with} \quad n>10\,,\qquad \text{for } \ell\neq 0\,, \tag{3.6}$$

where a_0 is a constant. In the presence of the singularity, a possible boundary condition is simply to require that the fields should not feel the singularity at all: $R_{m\ell}(0)=0$ disallowing solutions with $R_{m\ell}(0)\neq 0$ at $r=0$ as in [17]. Then the spectrum is bounded from below and continuous with only positive energies occurring [29, 30]. And the wavefunctions with $m\neq 0$ will be suppressed in most of the interior region, as will be cleared up later. For the massless graviton (i.e. $m=\ell=0$), eq. (3.3) admits the constant solution $R_{00}(r)=$ constant. However, this massless graviton state should be excluded, since it does not vanish at $r=0$. Hence, the inverse square law for gravity will not be recovered at long distance on a brane residing in the interior region.

Another possible boundary condition is the unitary boundary condition which was discussed in [27, 28]. The fact that the spacetime is geodesically incomplete would not matter provided that no conserved quantities are allowed to leak out through the boundary. Even though the singularity of the black 3-brane at $r=0$ is point-like and stronger than those considered in [27, 28], it is easy to see, following

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the same procedure as in refs. [27, 28], that such boundary condition allows the form of solution (3.5), that is, the flux through the singularity for the solutions having behaviors of (3.5) vanishes. Further, the constant solution for the massless graviton is not excluded by such boundary condition. Hence, the usual four-dimensional gravity will be reproduced at long distance, even though the spacetime has a naked singularity.

However, since we do not have a good understanding of the singularity, we will not be able to make any rigorous claim whether such boundary conditions are what we want. Thus, we will simply assume that the singularity is smoothed out by the true short-distance theory of gravity, namely, string theory. The singular source of the R-R field then will be look like a smooth 3-brane. Since these effects will only modify the metric close to the singularity, they only slightly change the shape of the wavefunctions. The massless graviton then is always allowed and the correct four-dimensional gravity is reproduced at long distance on a the 3-brane.

4. Gravity on the brane and mass hierarchy

It will be useful to work using the radial coordinate ρ of eq. (2.4) to see the effective gravity on a 3-brane residing in the interior region. The linearized equation (3.3) then can be written into the form of an analog non-relativistic Scarödinger equation by making a change of variable

$$\chi_{m\ell}(\rho) = K(\rho)\ R_{m\ell}(r(\rho))\,, \tag{4.1}$$

where

$$K(\rho) \equiv \left(\frac{r(\rho)}{r_H}\right)^{5/2} [-\Delta(r(\rho))]^{3/8} \simeq \begin{cases} \left(\dfrac{6\rho}{r_H}\right)^{1/6} & \text{for} \quad \rho \ll r_H \\ \left(\dfrac{\rho}{r_H}\right)^{-3/2} & \text{for} \quad \rho \gg r_H \end{cases}. \tag{4.2}$$

Then the resulting analog Schrödinger equation is given by

$$\left[-\frac{\partial^2}{\partial\rho^2} + V_{sch}(\rho) + V_{cfl}(\rho)\right] \chi_{m\ell}(\rho) = m^2\ \chi_{m\ell}(\rho)\,, \tag{4.3}$$

where $V_{sch}(\rho)$ is the analog non-relativistic quantum mechanical potential:

$$V_{sch}(\rho) \equiv \frac{K''(\rho)}{K(\rho)} = \frac{5}{4r_H^2}\frac{3y(\rho)^8 + 4y(\rho)^4 - 4}{y(\rho)^{10}}\sqrt{\frac{1}{y(\rho)^4} - 1} \tag{4.4}$$

$$\simeq \begin{cases} -\dfrac{5}{36}\dfrac{1}{\rho^2} & \text{for} \quad \rho \ll r_H \\ \dfrac{15}{4}\dfrac{1}{\rho^2} & \text{for} \quad \rho \gg r_H \end{cases}, \tag{4.5}$$

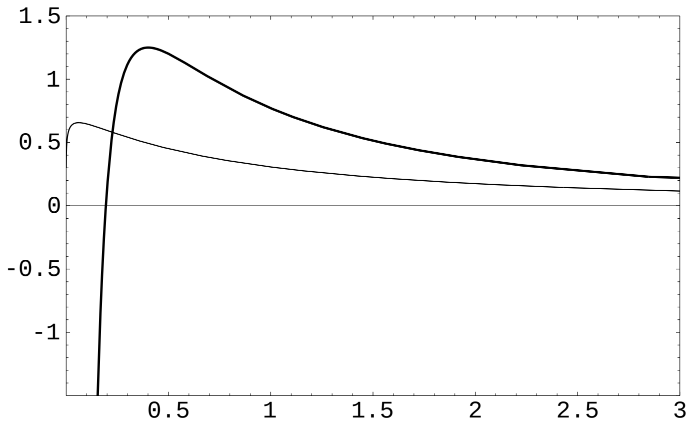

Figure 1: Plots of the Schrödinger potential $r_H^2 \cdot V_{sch}$ (thick solid line) and the massless graviton mode $K(\rho)$ (thin solid line) as functions of ρ/r_H.

where $y(\rho) \equiv r(\rho)/r_H$, and V_{cfl} is the repulsive centrifugal potential:

$$V_{cfl}(\rho) \equiv \frac{\ell(\ell+4)}{r_H^2}\frac{[y(\rho)^{-4}-1]^{1/2}}{y(\rho)^2} \simeq \begin{cases} \dfrac{\ell(\ell+4)}{r_H^2}\left(\dfrac{r_H}{6\rho}\right)^{2/3} & \text{for} \quad \rho \ll r_H \\ \dfrac{\ell(\ell+4)}{\rho^2} & \text{for} \quad \rho \gg r_H \end{cases} . \tag{4.6}$$

All of the important physics follows from a qualitative analysis of these potentials. The full form of the Schrödinger potential $V_{sch}(\rho)$ is sketched in figure 1. We see that the potential near the singularity approaches a negative infinity as $\sim -\rho^{-2}$. However, the negative infinity of the potential should be cut off at distances $\rho \sim l_s$ after smoothing out the singularity. Without a detailed understanding of the theory in such strongly gravitating region, we will not know the shape of the potential and so the shape of the wavefunctions near the origin. In [17], a model which modify the metric within the distance $\rho \sim l_s$ was presented to get an indication of the sort of modifications that can be expected once stringy corrections are taken into account. As we will see, however, such models are not needed in analyzing the wavefunctions in the interior region, because the amplitude of the wavefunction near the origin is insensitive to the detailed shape of the potential near the origin.

A bump results from the AdS spacetime near the horizon. The potential has a maximum value $5/4r_H^2$ at $\rho \approx 0.40r_H$ and is zero at $\rho \approx 0.19r_H$ showing that the central region of the potential is smaller than the AdS scale r_H^{-1}. The potential also approaches zero as $\sim \rho^{-2}$ near the horizon at $\rho = \infty$. For s-wave modes with $\ell = 0$, the potential in the near horizon region is identical to that of the original Randall-Sundrum model. While wave modes with $\ell \neq 0$ will see the additional repulsive centrifugal potential and will be much more suppressed near the origin than s-wave modes.

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The zero mode solution is easily identified from eq. (4.3) as $\chi_{00}(\rho) = a_{00}K(\rho)$, which corresponds to the constant zero-mode solution of eq. (3.3). Here, a_{00} ($\equiv (\pi^3 r_H^6/3)^{-1/2}$) is the normalization factor. The function $K(\rho)$ is plotted in figure 1. Clearly, the graviton is localized in the central region of the potential, even though $K(\rho)$ rapidly approaches zero as $\sim (\rho/r_H)^{1/6}$ at the very near of the origin when the regularization is not imposed. This seems to show that the gravity is completely decoupled from the δ-function like source at the singularity. The function $K(r)$, however, should be modified near the origin when the singularity is regularized. Since $K(l_s) \sim (l_s/r_H)^{1/6}$ at $\rho \sim l_s$, the function $K(r)$ does not significantly change in the central region $l_s \lesssim \rho \lesssim 0.4 r_H$, even for the large difference of order $l_s/r_H \sim 10^{-5}$. Hence, the central region looks like a one-sided Randall-Sundrum domain wall embedded in AdS_5 upon dimensional reduction. And the continuum modes in the near horizon region are given by a linear combination of Bessel functions

$$\chi_{m\ell}(\rho) = \sqrt{\rho}\left[a_{m\ell}Y_{2+\ell}(m\rho) + b_{m\ell}J_{2+\ell}(m\rho)\right], \tag{4.7}$$

where $a_{m\ell}$ and $b_{m\ell}$ are m-dependent coefficients to be determined by normalization and boundary conditions. The bump resulting from the AdS spacetime causes the continuum modes with masses $m \ll r_H^{-1}$ to be suppressed in the central region of the potential (close to the singularity). It is difficult to find quantitative behaviors of the continuum modes in the central region of the potential directly solving the Schrödinger equation. However, when the central region of the potential is localized within the AdS curvature length scale, the behaviors of the continuum modes can be determined from the asymptotic solution (4.7) using some matching conditions, as discussed in [30]. Following the same procedure as in [30], we obtain the value of the radial wavefunction in the central region up to the leading order in m,

$$\chi_{m\ell}(\rho) \sim \left(\frac{m}{r_H^{-1}}\right)^{\ell+1/2}. \tag{4.8}$$

While the continuum modes with $m > r_H^{-1}$ will sail over the potential and will be unsuppressed in the central region. In order to see the physics more explicitly, suppose a stack of D3-brane sitting at the origin which is strongly gravitating or a probe 3-brane residing at a point in the central region $l_s \lesssim \rho \lesssim 0.4 r_H$ which is very light. The gravitational potential $U(|\vec{x}|)$ of a test particle with mass m^* on the brane then is given by

$$\frac{U(|\vec{x}|)}{m^*} \sim G_4\frac{1}{|\vec{x}|} + \frac{1}{M_*^8}\sum_{\ell}\int_{m\neq 0}\frac{dm}{r_H^{-1}}\left(\frac{m}{r_H^{-1}}\right)^{\delta}|\chi_{m\ell}(\rho)|^2\frac{e^{-m|\vec{x}|}}{|\vec{x}|}, \tag{4.9}$$

where G_4 ($\sim |\chi_{00}(\rho)|^2/M_*^8$) is the four-dimensional gravitational constant and $|\vec{x}|$ is the distance from the test particle on the brane. δ is a power of proper measure for sums over the continuum modes and has ~ 0 for small ℓ and ~ 5 for large ℓ. For large distances $|\vec{x}| \gg r_H$, the first term generated by the four-dimensional graviton bound state is dominant and, using eq. (4.8), we get

$$U(|\vec{x}|) \sim G_4 \frac{m^*}{|\vec{x}|} \left[1 + \frac{1}{2} \left(\frac{r_H}{|\vec{x}|} \right)^2 + \sum_{\ell \neq 0} \left(\frac{r_H}{|\vec{x}|} \right)^{\delta+2+2\ell} \right]. \tag{4.10}$$

Since modes with $m \ll r_H^{-1}$ only contribute to the long distant gravity, modes with small ℓ are dominant and so $\delta \sim 0$. The leading correction behave like $(r_H/|\vec{x}|)^2$, as in the original Randall-Sundrum model, rather than $(r_H/|\vec{x}|)^7$, which may usually be expected to be for Randall-Sundrum type models with 6 extra dimensions. On the other hand, for distances $|\vec{x}| \ll r_H$, the continuum modes with $m \gg r_H^{-1}$ are dominant and has unsuppressed wave functions in the central region. Modes with large ℓ largely contribute to the short distance gravity and then $\delta \sim 5$ in the integrand of eq. (4.9). Therefore, for $|\vec{x}| \ll r_H$, we get the 10-dimensional gravitational potential

$$U(|\vec{x}|) \sim \frac{1}{M_*^8} \frac{m^*}{|\vec{x}|^7}. \tag{4.11}$$

Hence, the behavior of the gravity on the brane is exactly what we would expect by interpreting r_H as a "compactification radius", concretely supporting the interpretation of the horizon size as the compactification scale. That is, the interior region of the D3-brane looks like the four-dimensional spacetime producted by six extra dimensions compactified to the size of the horizon. Indeed, to an observer living outside the horizon, this interpretation seems correct in that the interior region occupies only a finite part of volume $\sim r_H^6$ of the transverse space. The apparent infinite extension of the interior region is simply the result of the warping of the extra dimensions by the gravity produced by the D3-brane.

The physics on a brane residing in the central region interpolates between the Randall-Sundrum one and the large extra dimensional scenario relying on the size r_H of the horizon. In case that the string scale is of the order of the four-dimensional Planck scale, i.e. $l_s^{-1} \sim M_{pl}$, the size of the horizon is to be of the order of the string scale. Thus, at low energy below the Planck scale, the extra space is reduced effectively to a one-dimensional space. Consequently, the central region looks like a one-sided Randall-Sundrum brane embedded in AdS_5 bulk spacetime. Since the information on the compactified sphere S^5 is reflected only through the extremely suppressed contribution from modes with $\ell \neq 0$ and moduli fields associated with the radius r_H of S^5 have masses of the order of the string scale, the phenomenology on the brane world is imperceptibly different from that of the original Randall-Sundrum scenario. This is what is expected from the supergravity domain wall solution of ref.[17] to the dimensionally reduced type IIB theory on S^5. However, this limit does not seem to be reliable because the supergravity description of the black 3-brane is inappropriate when $r_H \sim l_s$, as discussed in the section 2.

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When the horizon size is much larger than the string scale, i.e. $r_H \gg l_s$, the supergravity description of the interior region is valid except very near of the singularity. In this regime, at least in gravity side, the phenomenology seems to be very similar to that of the conventional large extra dimension scenarios with size r_H. Since the effective Planck scale is determined via the relation $M_{pl}^2 = r_H^6/(24\pi^3 g_s^2 l_s^8)$ and the size of the horizon is given by $r_H^4 = 4\pi l_s^4 g_s N$, the hierarchy between the string scale l_s^{-1} and the four-dimensional Planck scale M_{pl} can be provided with the R-R charge of the amount of $N = \left(2\pi^{3/2} l_s^2 g_s^{1/2} M_{pl}^2\right)^{2/3}$, that is,

$$\frac{M_{pl}}{l_s^{-1}} \sim N^{3/4}\,, \tag{4.12}$$

where we have taken the string coupling, g_s, to be of the order of one. If we naively assume that the string scale is of the order of the weak scale, i.e. $l_s^{-1} \sim m_{EW}$, then the magnitude of the R-R charge needed to generate the hierarchy is $N \sim 10^{22}$ and then the size of the horizon is $\sim 10^{-12}$cm. Hence, a D3-brane with large R-R charge, $N \sim 10^{22}$, provides a setup for a dynamical determination of the large hierarchy between the Planck scale and the string scale of the order of the weak scale. Moreover, the compactification scale is now determined by the R-R charge of the D3-brane and its stabilization is guaranteed by the conserved nature of the R-R charge.

5. Hawking radiation and small cosmological constant

Hitherto, we have assumed that our world brane is residing in the interior region of an extremal D3-brane. However, it seems that we are living in a non-extremal D3-brane rather than the extremal one because the present entropy density of our universe is not zero even though it is small, while the extremal D3-brane has zero entropy. In this section, we will consider the non-extremal version of the D3-brane and briefly discuss a few issues associated to the brane world scenarios.

In string frame and with a Schwarzschild-type radial coordinate r, the metric of the non-extremal D3-brane can be written as [1]

$$ds^2 = \frac{\Delta_+(r)}{\sqrt{\Delta_-(r)}}\left(-dt^2 + \frac{\Delta_-(r)}{\Delta_+(r)}d\vec{x}^2\right) + \frac{dr^2}{\Delta_+(r)\Delta_-(r)} + r^2 d\Omega_5^2\,, \tag{5.1}$$

where $\Delta_\pm(r) \equiv 1 - r_\pm^4/r^4$ and $r_+ > r_-$. This metric has a nondegenerate, nonsingular outer horizon at $r = r_+$. At $r = r_-$, one encounters an inner horizon, which, however, coincides with a curvature singularity. The singularity is covered with the outer horizon and so it does not matter to one who lives outside the outer horizon $r > r_+$. Then it can be regarded as a black hole.

Clearly the metric (5.1), however, is appropriate only for $r > r_-$. A solution which is appropriate for $r < r_-$ can simply be obtained by replacing $\Delta_\pm(r)$

with $\tilde{\Delta}_{\pm}(r) \equiv r_{\pm}^4/r^4 - 1$ for $r < r_-$ from the metric (5.1). This interior solution interpolates between two singularities at $r = 0$ and $r = r_-$. The singularity at $r = 0$ is due to the δ-function like source of the R-R self-dual 5-form field and it could be regarded as a stack of D3-branes. While, the singularity at $r = r_-$ is actually deriven by the non-linearity of the Einstein equations and disappears in the extremal limit, $r_- = r_+ = r_H$. It is clear that the interior (the exterior) solution reduce to the interior (the exterior) solution of eq. (2.1) in the extremal limit. The singularity at $r = r_-$ completely disappears as $r_- = r_+$ and the Poincaré invariance in the x^μ-direction is recovered.

In the non-extremal case, the interior solution is bounded by the naked singularity. That is, the curvature singularity sits at some finite proper distance from the brane on which visible sector matter exists. Such naked spacetime singularity may be interpreted as the boundary of the extra dimension.[2] The proper interpretation of this singularity will likely be crucial to understanding the physics on the world brane. For example, the singularity may force the short-distance properties of quantum gravity to become relevant to the physics at long distances. However, since we do not have complete understandings about the singularity and the quantum gravity (or stringy effects on such large curvature background), we will assume that the singularity is smoothed out by some stringy effects. The spacetime then could be extended across the singularity into a region where a weakely coupled Einstein decription is valid again. There would be various possibilities of the extension across the horizon. Here, we will simply assume that the interior region is glued by the exterior region of the metric (5.1) at the resolved singularity. The spacetime then has a causal structure similar to that of the non-extremal Reissner-Nordström black hole. In this case there would be no 4-dimensional gravity at long distances because the interior region is not closed and the extra dimensions have an inifinte volume.[3] However, in the extremal limit the interior region is closed with a finite volume at the singularity, which is now regular and pushed to infinite proper distance. In this case, the extra dimensions are effectively compactified as shown in the previous sections, and we should get 4-dimensional gravity at long distances.

As mentioned at the beginning of this section, we may live in the interior region of a near-extremal D3-brane. Then, how much extremal can the D3-brane be it to be our world brane? Defining the parameters r_+ and r_- as $\delta^4 \equiv r_+^4 - r_-^4$, the

[2]Similar constructions have appeared in variants of the Randall-Sundrum construction [10]–[15]. And it was proposed that the singularities could help solve the cosmological constant problems [10]–[14]. However, in those constructions a crucial difference from the present one is that the geometry with the four-dimensional Poincaré invariance in the x^μ-direction has the naked singularity. While, in the spacetime described by the metric (5.1), the singularity disappears in the limit that the Poincaré invariance in x^μ-direction is recovered.

[3]However, we need a careful investigation about the role of the spacelike region between the inner horizon and the outer horizon for the effective 4-dimensional gravity.

Hawking temperature and the Bekenstein-Hawking entropy (per unit volume in the $\vec{x}$-direction) are, respectively, given by

$$T_{\rm H} = \frac{\delta}{\pi r_+^2} \qquad \text{and} \qquad s_{BH} \sim \frac{r_+^2 \delta^3}{g_s^2 l_s^8}\,, \tag{5.2}$$

up to the numerical factor of the order of one. The extremal solution has a degenerate horizon $r_+ = r_-$, and zero Bekenstein-Hawking entropy. The Hawking temperature of the extremal brane is also zero. In the near extremal limit, the entropy is

$$s_{BH} \sim \frac{r_H^{17/4} \epsilon^{3/4}}{g_s^2 l_s^8}\,, \tag{5.3}$$

where $\epsilon \equiv r_+ - r_-$. Notice that the black brane entropy is an extremely enormous number even for the near extremal black brane with $\epsilon \sim l_s$, e.g. $s_{BH} \sim 10^{73}/\mathrm{cm}^3$ as $l_s \sim m_{EW}^{-1}$ and $r_H \sim 10^{-12}$cm. On the other hand, the present entropy density of our universe is at most $\sim 10^3/\mathrm{cm}^3$. This seems to imply that our world brane can easily be embedded in the interior region of a extremely near-extremal D3-brane.

Since a non-extremal black brane has nonzero Hawking temperature and Hawking radiates, it evolves into an extremal black brane. Notice that the curvature singularity at $r = r_-$ is resolved through this process. As discussed in ref. [25], we may take this as a possible mechanism of resolution of various problems associated with the brane world scenario. For example, for the metric (5.1) the bulk curvature leads to violation of the SO(3, 1) isometry, in the D3-brane worldvolume direction, observed in our brane, so there must be some mechanism to flatten the bulk in the absence of some symmetry which protect the SO(3, 1) isometry [31]. Clearly, the bulk curvature can be diluted via the Hawking radiation because in the extremal limit the bulk curvature vanishes and the SO(3, 1) isometry of the D3-brane worldvolume is recovered.

In general, excitations of the extremal black p-brane would correspond to either non-Ricci flat $\hat{g}_{\mu\nu} = \hat{g}_{\mu\nu}(x)$ such that $\hat{R}(\hat{g}) \neq 0$, or even to depend on the extra dimension coordinates $\hat{g}_{\mu\nu} = \hat{g}_{\mu\nu}(x, y)$. Then the corresponding non-extremal metric would have different forms from the metric eq. (5.1). Even though we don't have such metric yet, we expect that the spacetime structure will be close to that of metric (5.1) and so the excitations will be diluted via Hawking radiation. Hence, with the same argument as in ref. [25] the observed extremely small cosmological constant could be explained, provided that our world brane is embedded in the interior region of a very near-extremal black brane. The change of the vacuum energy density of the brane which may break the Poincaré invariance of the D3-brane worldvolume will be diluted through the Hawking radiation process. In the same way, the observed flatness and approximate Lorentz invariance of our world brane could be also explained.

6. Conclusions

We have shown that the interior region inside the horizon of the D3-brane possesses all of features needed for a realistic brane world scenario which interpolates between the Randall-Sundrum scenario and the large extra dimension scenario, provided that the singularity at the origin is smoothed out. In this picture, the horizon size can be interpreted as the compactification size in that the four-dimensional Planck scale M_{pl} is determined by the 10-dimensional Planck scale $M_* \sim l_P^{-1} = g_s^{-1/4} l_s^{-1}$ and the size of the horizon r_H via the familiar relation $M_{pl}^2 \sim M_*^8 r_H^6$ and the effective gravity on a 3-brane residing in the interior region behaves as expected in a world with six extra dimensions compactified with size r_H.

The most important consequence of the above picture seems that the macroscopically large compactification scale can be derived from the underlying string theory that has only one physical scale, i.e. the string scale l_s. That is, the size of the large extra dimensions is provided by the large R-R charge of the D3-brane and its stabilization is strictly guaranteed from the charge conservation law. Then, the hierarchy between the Planck scale and the string scale is dynamically determined by the magnitude of the R-R charge N carried by the D3-brane without introducing any additional scale.

In this picture, the finetuning required in the original RS setup in order to ensure the vanishing of the four-dimensional cosmological constant could be eliminated. The vacuum energy density of the brane can be diluted through the Hawking radiation and the observed extremely small cosmological constant can be explained provided that our world brane is a very near-extremal D3-brane. The observed flatness and approximate Lorentz invariance of our world brane could be explained with the same token.

Acknowledgments

We would like to thank Sangheon Yi, Jong Dae Park, Hyun Min Lee, O-Kab Kwon and Soo-Jong Rey for useful discussions. The work is supported in part by Korea Research Foundation (KRF-2000-015-BP0072), and by the BK21 project of Ministry of Education.

References

[1] G.T. Horowitz and A. Strominger, *Black strings and p-branes*, *Nucl. Phys.* **B 360** (1991) 197.

[2] J. Polchinski, *Tasi lectures on D-branes*, `hep-th/9611050`.

[3] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315]; *Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity*, *Phys. Rev.* **D 59** (1999) 086004 [hep-ph/9807344].

[4] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *New dimensions at a millimeter to a Fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398];
I. Antoniadis and C. Bachas, *Branes and the gauge hierarchy*, *Phys. Lett.* **B 450** (1999) 83 [hep-th/9812093].

[5] G. Shiu and S.H. H. Tye, *Tev scale superstring and extra dimensions*, *Phys. Rev.* **D 58** (1998) 106007 [hep-th/9805157];
Z. Kakushadze and S.H. H. Tye, *Brane world*, *Nucl. Phys.* **B 548** (1999) 180 [hep-th/9809147];
L.E. Ibáñez, C. Muñoz and S. Rigolin, *Aspect of type I string phenomenology*, *Nucl. Phys.* **B 553** (1999) 43 [hep-ph/9812397].

[6] K. Benakli, *Phenomenology of low quantum gravity scale models*, *Phys. Rev.* **D 60** (1999) 104002 [hep-ph/9809582];
G. Aldazabal, L.E. Ibáñez and F. Quevedo, *A D-brane alternative to the MSSM*, *J. High Energy Phys.* **02** (2000) 015 [hep-ph/0001083].

[7] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[8] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[9] V.A. Rubakov and M.E. Shaposhnikov, *Extra space-time dimensions: towards a solution to the cosmological constant problem*, *Phys. Lett.* **B 125** (1983) 139.

[10] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, *A small cosmological constant from a large extra dimension*, *Phys. Lett.* **B 480** (2000) 193 [hep-th/0001197].

[11] S. Kachru, M. Schulz and E. Silverstein, *Bounds on curved domain walls in 5D gravity*, *Phys. Rev.* **D 62** (2000) 085003 [hep-th/0002121].

[12] C. Csaki, J. Erlich, C. Grojean and T. Hollowood, *General properties of the self-tuning domain wall approach to the cosmological constant problem*, *Nucl. Phys.* **B 584** (2000) 359 [hep-th/0004133].

[13] S. Forste, Z. Lalak, S. Lavignac and H.P. Nilles, *A comment on self-tuning and vanishing cosmological constant in the brane world*, *Phys. Lett.* **B 481** (2000) 360 [hep-th/0002164]; *The cosmological constant problem from a brane-world perspective*, *J. High Energy Phys.* **09** (2000) 034 [hep-th/0006139].

[14] J.-W. Chen, M.A. Luty, E. Ponton, *A critical cosmological constant from millimeter extra dimensions*, *J. High Energy Phys.* **09** (2000) 012.

[15] C. Grojean, J. Cline and G. Servant, *Supergravity inspired warped compactifications and effective cosmological constants*, *Nucl. Phys.* **B 578** (2000) 259 [hep-th/9910081];
G.T. Horowitz, I. Low and A. Zee, *Self-tuning in an outgoing brane wave model*, *Phys. Rev.* **D 62** (2000) 086005 [hep-th/0004206];
Z. Kakushadze, *Localized (super)gravity and cosmological constant*, *Nucl. Phys.* **B 589** (2000) 75 [hep-th/0005217];
C. Schmidhuber, *AdS_5 and the $4D$ cosmological constant*, *Nucl. Phys.* **B 580** (2000) 140 [hep-th/9912156]; *Micrometer gravitinos and the cosmological constant*, *Nucl. Phys.* **B 585** (2000) 385 [hep-th/0005248];
Chuan-Jie Zhu, *A self-tuning exact solution and the non-existence of horizons in 5D gravity-scalar system*, *J. High Energy Phys.* **06** (2000) 034 [hep-th/0005230];
C.P. Burgess, R.C. Myers and F. Quevedo, *A naturally small cosmological constant on the brane?*, *Phys. Lett.* **B 495** (2000) 384 [hep-th/9911164];
V. Barger, T. Han, T. Li, J.D. Lykken and D. Marfatia, *Cosmology and hierarchy in stabilized warped brane models*, *Phys. Lett.* **B 488** (2000) 97 [hep-ph/0006275];
P. Binetruy, J.M. Cline and C. Grojean, *Dynamical instability of brane solutions with a self-tuning cosmological constant*, *Phys. Lett.* **B 489** (2000) 403 [hep-th/0007029];
H. Collins and B. Holdom, *The cosmological constant and warped extra dimensions*, *Phys. Rev.* **D 63** (2001) 084020 [hep-th/0009127];
P. Brax and A.C. Davis, *Cosmological solutions of supergravity in singular spaces*, *Phys. Lett.* **B 497** (2001) 289 [hep-th/0011045];
N. Tetradis, *On brane stabilization and the cosmological constant*, hep-th/0012106.

[16] S.P. de Alwis, *Brane world scenarios and the cosmological constant*, *Nucl. Phys.* **B 597** (2001) 263 [hep-th/0002174];
S.P. de Alwis, A.T. Flournoy and N. Irges, *Brane worlds, the cosmological constant and string theory*, *J. High Energy Phys.* **01** (2001) 027 [hep-th/0004125].

[17] M. Cvetič, H. Lu and C.N. Pope, *Localized gravity in the singular domain wall background?*, hep-th/0002054.

[18] M. Cvetič, H. Lu and C.N. Pope, *Domain walls and massive gauged supergravity potentials*, *Class. and Quant. Grav.* **17** (2000) 4867 [hep-th/0001002]; *Domain walls with localised gravity and domain-wall/QFT correspondence*, *Phys. Rev.* **D 63** (2001) 086004 [hep-th/0007209].

[19] M.S. Bremer, M.J. Duff, H. Lu, C.N. Pope and K.S. Stelle, *Instanton cosmology and domain walls from M-theory and string theory*, *Nucl. Phys.* **B 543** (1999) 321 [hep-th/9807051].

[20] M.J. Duff, J.T. Liu and K.S. Stelle, *A supersymmetric type IIB Randall-Sundrum realization*, hep-th/0007120.

[21] K. Behrndt and M. Cvetič, *Supersymmetric domain wall world from $D = 5$ simple gauged supergravity*, *Phys. Lett.* **B 475** (2000) 253 [hep-th/9909058];
Anti-de Sitter vacua of gauged supergravities with eight supercharges, *Phys. Rev.* **D 61** (2000) 101901 [hep-th/0001159].

[22] J. Lykken and L. Randall, *The shape of gravity*, *J. High Energy Phys.* **06** (2000) 014 [hep-th/9908076].

[23] C.S. Chan, P.L. Paul and H. Verlinde, *A note on warped string compactification*, *Nucl. Phys.* **B 581** (2000) 156 [hep-th/0003236];
B.R. Greene, K. Schalm and G. Shiu, *Warped compactifications in M and F theory*, *Nucl. Phys.* **B 584** (2000) 480 [hep-th/0004103].

[24] C.S. Chan, P.L. Paul and H. Verlinde, *A note on warped string compactification*, *Nucl. Phys.* **B 581** (2000) 156 [hep-th/0003236].

[25] S.-H. Moon, S.-J. Rey and Y. Kim, *Global black p-brane world: a new approach to stable mass hierarchy*, hep-th/0012165, to appear in *Nucl. Phys.* **B**.

[26] G.W. Gibbons, G.T. Horowitz and P.K. Townsend, *Higher dimensional resolution of dilatonic black hole singularities*, *Class. and Quant. Grav.* **12** (1995) 297 [hep-th/9410073].

[27] A.G. Cohen and D.B. Kaplan, *Solving the hierarchy problem with noncompact extra dimensions*, *Phys. Lett.* **B 470** (1999) 52 [hep-th/9910132].

[28] M. Gremm, *Four-dimensional gravity on a thick domain wall*, *Phys. Lett.* **B 478** (2000) 434 [hep-th/9912060].

[29] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, *Modeling the fifth dimension with scalars and gravity*, *Phys. Rev.* **D 62** (2000) 046008 [hep-th/9909134].

[30] C. Csaki, J. Erlich, T.J. Hollowood and Y. Shirman, *Universal aspects of gravity localized on thick branes*, *Nucl. Phys.* **B 581** (2000) 309 [hep-th/0001033].

[31] D.J.H. Chung, E.W. Kolb and A. Riotto, *Extra dimensions present a new flatness problem*, hep-ph/0008126.